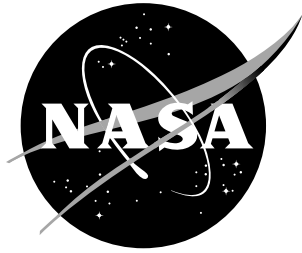


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Sensitivity Analysis for Coupled Aero-structural Systems

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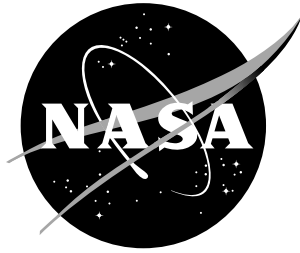
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Sensitivity Analysis for Coupled Aero-structural Systems

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Abstract

A novel method has been developed for calculating gradients of aerodynamic force and moment coefficients for an aeroelastic aircraft model. This method uses the Global Sensitivity Equations (GSE) to account for the aero-structural coupling, and a reduced-order modal analysis approach to condense the coupling bandwidth between the aerodynamic and structural models. Parallel computing is applied to reduce the computational expense of the numerous high fidelity aerodynamic analyses needed for the coupled aero-structural system. Good agreement is obtained between aerodynamic force and moment gradients computed with the GSE/modal analysis approach and the same quantities computed using brute-force, computationally expensive, finite difference approximations. A comparison between the computational expense of the GSE/modal analysis method and a pure finite difference approach is presented. These results show that the GSE/modal analysis approach is the more computationally efficient technique if sensitivity analysis is to be performed for two or more aircraft design parameters.

Nomenclature

C_D	drag coefficient
C_L	lift coefficient
C_{M_o}	pitching moment coefficient (about y-axis)
CFD	computational fluid dynamics
CSM	computational structural mechanics
E	Young's modulus
\mathbf{F}	vector of aerodynamic loads
GSE	Global Sensitivity Equations
I	area moment of inertia
\mathbf{I}	identity matrix
\mathbf{K}	finite element stiffness matrix
L	beam length
LE	leading edge
LSM	Local Sensitivity Matrix
LSV	Local Sensitivity Vector

\mathbf{M}	finite element mass matrix
MDO	multidisciplinary design optimization
OML	outer mold line
\mathbf{q}	vector of mode shape scale factors
TSV	Total Sensitivity Vector
t/c	thickness-to-chord ratio
\mathbf{X}	vector of independent design parameters
α	angle-of-attack
δ	beam tip deflection
Δ	vector of structural deflections
λ	eigenvalue from modal analysis
Λ	diagonal matrix of eigenvalues
Λ_{LE}	inboard leading edge sweep angle
ϕ	eigenvector from modal analysis
Φ	column matrix of eigenvectors

1 Introduction

In Paul Rubbert's 1994 AIAA Wright Brother Lecture [1] he states a vision for the future of the aircraft design process as follows,

"My vision is to be able to carry out the detailed aerodynamic design of any portion of an airplane within a handful of days at most, and to do it in concert with the loads engineer, the structural designer, the systems person and the manufacturing expert sitting side by side in the same room, with computer systems that talk well with one another."

Implicit in this statement is that the computational models used in aircraft design accurately capture the important physical phenomena of interest. That is, high fidelity analysis models are available for aerodynamic analysis, structural analysis, and for the other aircraft design disciplines. Rubbert's vision that the computer systems "talk well with one another" supposes that a capability exists to exchange the discipline-specific high fidelity analysis data among the different engineering disciplines. The communication links among the engineering disciplines are necessary to perform the sensitivity analyses needed to quantify the impact of design perturbations (e.g., changes in wing thickness) on the performance of the

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aircraft (e.g., aerodynamic performance, structural weight, manufacturing costs). Such sensitivity studies are the foundation for improving, or optimizing, an aircraft design.

Progress toward Rubbert’s vision is occurring gradually as high fidelity analysis tools, such as Euler/Navier-Stokes computational fluid dynamics (CFD) solvers and finite element computational structural mechanics (CSM) codes, are employed increasingly early in the aircraft design process. However, there are several impediments to Rubbert’s vision of efficient and accurate computing in aircraft design. One of these impediments is the computational burden incurred when expensive CFD and CSM codes are employed in the analysis of coupled aero-structural systems such as modern transport and fighter aircraft. Analyzing these coupled systems typically requires the repeated use of CFD and CSM codes to obtain an accurate solution. Thus, the computational expense quickly mounts if many coupled aero-structural analyses must be performed for sensitivity analysis and/or optimization.

The focus of this work is the development of a computational method that permits the rapid evaluation of sensitivity derivatives (gradients) for a coupled aero-structural system. This study employs the Global Sensitivity Equation (GSE) method developed by Sobieski [2] which provides a mathematical expression for the total sensitivity derivatives of a general coupled system. For the aero-structural system considered here, the GSE method requires the computation of interdisciplinary coupling terms (partial derivatives) between the aerodynamic and structural models. To reduce the number of partial derivatives terms in the GSE, structural deflections are approximated using a superposition of structural mode shapes (basis vectors). Additional computational savings are realized by using coarse grained parallel computing for the CFD evaluations needed to compute some of the partial derivatives in the GSE.

The combined GSE/modal analysis approach developed in this study enables an aircraft design engineer to perform a sensitivity analysis of a coupled aero-structural system in a manner that is more efficient than using traditional finite difference methods. In addition, the GSE/modal analysis approach employs the CFD and CSM solvers as black-boxes. Thus, virtually any CFD or CSM code may be used with this sensitivity analysis method.

The GSE/modal analysis approach is intended for use in the preliminary phase of aircraft design when both detailed CFD and CSM (finite element) models have been created for an aircraft. The use of a linear superposition of mode shapes to represent struc-

tural deflections of a finite element model limits the GSE/modal analysis approach to the exploration of perturbations of an existing aircraft design. That is, one assumption of the GSE/modal analysis method is that the natural frequencies and mode shapes remain essentially unchanged for small perturbations in the aircraft design parameters. For this reason, this approach is not intended for use in the conceptual phase of aircraft design when major configuration choices have yet to be finalized.

In this study, the Langley-developed CFD solver CFL3D [3] is used for aerodynamic analysis and the commercial CSM solver GENESIS [4] is used for structural analysis. The aircraft examined here is a generic supersonic transport configuration. The parametric model for this aircraft contains 104 design variables (64 planform and airfoil variables, 40 structural variables). Gradients of aerodynamic force and moment coefficients are computed for three of these variables using the GSE/modal analysis approach. Validation of the accuracy of the GSE/modal analysis gradients is performed through comparisons to gradients computed using a pure finite difference approach.

The remainder of this paper is arranged as follows. Section 2 contains some background information on related research using GSE methods and reduced-order modal analysis methods in computational aeroelasticity. Section 3 covers the mathematics of the GSE formulation and the modal analysis methods employed in this study. Section 4 contains a simple problem involving a cantilever beam to demonstrate the GSE/modal analysis method. Sections 5 and 6 cover the modeling and aeroelastic analysis of a supersonic transport aircraft, along with results obtained from applying the GSE/modal analysis method to calculate sensitivities for this aircraft model. In Section 7 there is a comparison of the computational expense of the GSE/modal analysis approach and the traditional finite difference approach for calculating sensitivity derivatives. A summary of this work is contained in Section 8.

2 Background

This study builds on the past research efforts of Barthelemy et al [5] and Dovi et al [6] who employed the Global Sensitivity Equations in design optimization of a supersonic transport aircraft. Related research by Kapania et al [7] and Eldred et al [8] used a variation of the GSE method for the sensitivity analysis of a simple forward-swept wing model. These studies employed inexpensive low fi-

delity analysis methods such as linear aerodynamics (panel codes) and equivalent plate structural models (ELAPS [9, 10]). These inexpensive methods permitted the use of finite difference approximations to evaluate the interdisciplinary coupling derivatives in the GSE (described in Section 3).

The use of computationally expensive high fidelity analysis software in this study greatly increases the amount of interdisciplinary coupling derivatives in the GSE. That is, the number of coupling terms is related to the amount of force and deflection data exchanged between the aerodynamic and structural models. With the high fidelity CFD and CSM tools employed in this work, the number of coupling terms in the GSE is $\mathcal{O}(10^2 - 10^3)$. Thus, a pure finite difference approach for approximating the partial derivative coupling terms is not computationally affordable.

Fortunately, there has been considerable research in the aeroelasticity community in developing methods that simplify the coupling between aerodynamic and structural models. Numerous reduced-order modeling approaches are described in the survey papers by Friedmann [11], Livne [12], and Karpel [13]. Recent examples of the application of reduced-order modeling methods are found in the work of Raveh and Karpel [14] and Cohen and Kapania [15].

The reduced-order modeling approach followed in this study employs a linear superposition of basis vectors to approximate the structural deflections of an aircraft finite element model. Here, the basis vectors are supplied by a normal modes (eigenvalue) analysis of the finite element model. The number of basis vectors used in this study is $\mathcal{O}(10^1)$. One advantage of this reduced-order approach is that it supplies analytic expressions for some of the interdisciplinary coupling terms in the GSE. Another advantage of this approach is that it becomes computationally affordable to compute the remaining partial derivatives in the GSE using finite difference methods. The derivation and application of this GSE/modal analysis approach is described below.

3 Sensitivity Analysis

3.1 Global Sensitivity Equations

Consider a generic coupled aero-structural system depicted in Figure 1. The coupling between the aerodynamic and structural models is accomplished by calculating external loads, \mathbf{F} , on the aircraft structure along with the resulting structural deflections, Δ . The aerodynamic and structural data needed to compute \mathbf{F} and Δ are obtained from a variety of computer programs along with suitable pre- and

post-processing software to transfer the load and deflection data between the models.

The input into the aero-structural system is a vector of independent parameters \mathbf{X} . This vector contains aerodynamic design parameters, such as wing planform and shape parameters, as well as structural design parameters, such as internal rib and spar thicknesses. The output quantities from the coupled system include the aerodynamic load distribution on the deflected structure, \mathbf{F} , along with the force and moment coefficients C_L , C_D , and C_{M_o} . In addition, the output includes the final structural deflections, Δ , and the internal stresses in the structure.

For an aircraft design application, quantities of interest such as the aerodynamic force and moment coefficients are needed to ensure that the aircraft will satisfy the specified mission requirements. In addition, the gradients of these quantities are needed to determine the sensitivity of the aerodynamic performance to small perturbations in the design parameters. Analytic expressions for these gradients usually are not available and they are estimated using finite difference approximations. This requires a separate solution of the coupled system for each perturbation of \mathbf{X} ; a potentially prohibitive undertaking if either the aerodynamic model or the structural model is computationally expensive to evaluate.

Following the notation employed by Sobieski [2], the loads and deflection transfer in the aero-structural system are represented in functional form as

$$\mathbf{F} = \mathbf{F}(\mathbf{X}, \Delta), \quad (1)$$

and

$$\Delta = \Delta(\mathbf{X}, \mathbf{F}). \quad (2)$$

Differentiating Equations 1 and 2 with respect to the vector of independent parameters, \mathbf{X} , yields

$$\frac{d\mathbf{F}}{d\mathbf{X}} = \frac{\partial \mathbf{F}}{\partial \mathbf{X}} + \frac{\partial \mathbf{F}}{\partial \Delta} \frac{d\Delta}{d\mathbf{X}}, \quad (3)$$

and

$$\frac{d\Delta}{d\mathbf{X}} = \frac{\partial \Delta}{\partial \mathbf{X}} + \frac{\partial \Delta}{\partial \mathbf{F}} \frac{d\mathbf{F}}{d\mathbf{X}}. \quad (4)$$

Equations 3 and 4 are coupled and may be rearranged into a linear system of the form

$$\begin{bmatrix} \mathbf{I} & -\frac{\partial \mathbf{F}}{\partial \Delta} \\ -\frac{\partial \Delta}{\partial \mathbf{F}} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \frac{d\mathbf{F}}{d\mathbf{X}} \\ \frac{d\Delta}{d\mathbf{X}} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial \mathbf{F}}{\partial \mathbf{X}} \\ \frac{\partial \Delta}{\partial \mathbf{X}} \end{Bmatrix}. \quad (5)$$

Equation 5 is known as the Global Sensitivity Equation (GSE), which provides a convenient framework for grouping related terms in a system of coupled sensitivity equations. Olds [16] provides a useful lexicon for describing the components of the GSE.

Following Olds' approach, the matrix on the left side of Equation 5 is termed the Local Sensitivity Matrix (LSM) and it contains the partial derivatives of the aerodynamic loads and structural deflections with respect to each other. Note that the vector of independent parameters, \mathbf{X} , does not appear in the LSM. The vector on the right side of Equation 5 is termed the Local Sensitivity Vector (LSV). This vector contains the partial derivatives of the loads and deflections with respect to the independent parameters. In the LSV there are no coupling derivatives. The vector on the left side of Equation 5 is termed the Total Sensitivity Vector (TSV) and it contains the total sensitivity derivatives which are the unknown quantities in this system of equations.

While Equation 5 provides a simple expression for the interdisciplinary coupling of an aero-structural system, the partial derivative terms in Equation 5 may be particularly difficult to obtain. For example, consider the term $\partial\mathbf{F}/\partial\mathbf{\Delta}$ in Equation 5. The vector of aerodynamic forces applied to the structural model, \mathbf{F} , is computed using an expensive CFD code (assume for this example that suitable force calculation and interpolation methods are available). If one attempts to calculate $\partial\mathbf{F}/\partial\mathbf{\Delta}$ using a finite difference approximation, then a CFD code evaluation is required for each perturbation of the vector $\mathbf{\Delta}$. Even with parallel computing, this approach is not attractive when the length of $\mathbf{\Delta}$ is large, e.g., $\mathcal{O}(10^2 - 10^3)$, as would be typical for a finite element model used in the aircraft industry.

For this reason there is motivation to explore methods that may reduce the computational expense of computing the term $\partial\mathbf{F}/\partial\mathbf{\Delta}$. One approach to this problem is to represent the nodal deflections using a linear superposition of a set of basis functions. A version of such a reduced basis approach is employed in this study, where the basis functions are the mode shapes of the structural finite element model.

An important aspect of this work is that the reduced basis method used here is employed while treating the CFD and CSM codes as black boxes. That is, access to the source code of the CFD and CSM analysis software is not needed. While this approach may appear restrictive, it is a realistic model of the aircraft design industry in which both commercial software and legacy software are used in the aircraft analysis and design process. An attractive outcome of this restriction is that successful methods developed by treating the CFD and CSM codes as black boxes will be broadly applicable to aircraft design practices in industry.

3.2 Modal Analysis

In linear finite element structural analysis the vector of structural displacements, $\mathbf{\Delta}$, is found by solving the equation

$$\mathbf{F} = \mathbf{K}\mathbf{\Delta}, \quad (6)$$

where \mathbf{F} is the vector of applied loads and \mathbf{K} is the stiffness matrix assembled from the individual shape functions of the finite elements. Another common computation is a modal analysis of the finite element model. This analysis provides the natural frequencies of the structure along with the associated mode shapes. The modal analysis is performed by solving the eigenvalue problem

$$\mathbf{K}\phi = \mathbf{M}\phi\lambda, \quad (7)$$

where \mathbf{M} is the mass matrix of the finite element model, and ϕ is an eigenvector (mode shape) corresponding to a particular eigenvalue, λ , that satisfies the linear system.

Equation 7 is solved to obtain the desired number of eigenvalues and eigenvectors for the finite element model. These n eigenvalues are expressed using a diagonal matrix of the form

$$\mathbf{\Lambda} = [\lambda_1, \lambda_2, \dots, \lambda_n]\mathbf{I}, \quad (8)$$

where \mathbf{I} is an $n \times n$ identity matrix. The n eigenvectors are grouped using the matrix $\mathbf{\Phi}$ as

$$\mathbf{\Phi} = [\phi_1 \ \phi_2 \ \dots \ \phi_n], \quad (9)$$

where the n columns of $\mathbf{\Phi}$ correspond to the n eigenvalues in $\mathbf{\Lambda}$. Note that in this study the eigenvectors are scaled to meet the K-orthogonality and M-orthonormality conditions described by Bathe [17], such that

$$\mathbf{\Phi}^T \mathbf{K} \mathbf{\Phi} = \mathbf{\Lambda}, \quad (10)$$

and

$$\mathbf{\Phi}^T \mathbf{M} \mathbf{\Phi} = \mathbf{I}. \quad (11)$$

This scaling is performed internally in the CSM solver GENESIS [4] used in this study.

The mode shapes in $\mathbf{\Phi}$ are used to approximate the structural deflections, $\mathbf{\Delta}$, where

$$\mathbf{\Delta} = \mathbf{\Phi}\mathbf{q}, \quad (12)$$

and \mathbf{q} is a vector of unknown scale factors. The vector \mathbf{q} is computed using a least squares approach where

$$\mathbf{q} = (\mathbf{\Phi}^T \mathbf{\Phi})^{-1} \mathbf{\Phi}^T \mathbf{\Delta}. \quad (13)$$

Note that it is typical to use 10 – 30 mode shapes in this approximation. Thus, the length of \mathbf{q} can be much smaller than the length of the vector of structural deflections $\mathbf{\Delta}$.

To take advantage of this reduced basis approach involving \mathbf{q} , the Chain Rule is applied to the term $\partial\mathbf{F}/\partial\mathbf{\Delta}$ as follows

$$\frac{\partial\mathbf{F}}{\partial\mathbf{\Delta}} = \frac{\partial\mathbf{F}}{\partial\mathbf{q}} \frac{\partial\mathbf{q}}{\partial\mathbf{\Delta}}, \quad (14)$$

where, from Equation 13

$$\frac{\partial\mathbf{q}}{\partial\mathbf{\Delta}} = (\mathbf{\Phi}^T\mathbf{\Phi})^{-1}\mathbf{\Phi}^T. \quad (15)$$

Thus, Equation 14 becomes

$$\frac{\partial\mathbf{F}}{\partial\mathbf{\Delta}} = \frac{\partial\mathbf{F}}{\partial\mathbf{q}}(\mathbf{\Phi}^T\mathbf{\Phi})^{-1}\mathbf{\Phi}^T. \quad (16)$$

The advantage of this approach is that \mathbf{q} is $\mathcal{O}(10^1)$ whereas $\mathbf{\Delta}$ typically is $\mathcal{O}(10^2 - 10^3)$. It is much more reasonable to compute $\partial\mathbf{F}/\partial\mathbf{q}$ than $\partial\mathbf{F}/\partial\mathbf{\Delta}$ if using finite difference approximations.

The other partial derivative term in the Local Sensitivity Matrix of Equation 5 is $\partial\mathbf{\Delta}/\partial\mathbf{F}$. The exact value of this derivative is \mathbf{K}^{-1} , however, this matrix is not available to the user in many finite element analysis codes, particularly those that are commercially developed. For this reason, an approximation for $\partial\mathbf{\Delta}/\partial\mathbf{F}$ is needed. Using the modal analysis approach described above, it is possible to compose an explicit expression relating $\mathbf{\Delta}$ and \mathbf{F} . This is described below.

Substituting Equation 12 into Equation 6 yields

$$\mathbf{F} = \mathbf{K}\mathbf{\Phi}\mathbf{q}. \quad (17)$$

Pre- and post-multiplying this expression by $\mathbf{\Phi}^T$ and $\mathbf{\Phi}$, respectively, gives

$$\mathbf{\Phi}^T\mathbf{F}\mathbf{\Phi} = \mathbf{\Phi}^T\mathbf{K}\mathbf{\Phi}\mathbf{q}\mathbf{\Phi}. \quad (18)$$

The K-orthogonality condition in Equation 10 is used to simplify Equation 18 to

$$\mathbf{\Phi}^T\mathbf{F}\mathbf{\Phi} = \mathbf{\Lambda}\mathbf{q}\mathbf{\Phi}, \quad (19)$$

which reduces to

$$\mathbf{\Phi}^T\mathbf{F} = \mathbf{\Lambda}\mathbf{q}. \quad (20)$$

Rearranging this equation yields an expression for \mathbf{q} where

$$\mathbf{q} = \mathbf{\Lambda}^{-1}\mathbf{\Phi}^T\mathbf{F}. \quad (21)$$

Finally, substituting Equation 21 into Equation 12 gives

$$\mathbf{\Delta} = \mathbf{\Phi}\mathbf{\Lambda}^{-1}\mathbf{\Phi}^T\mathbf{F}. \quad (22)$$

With this explicit relationship between $\mathbf{\Delta}$ and \mathbf{F} , the analytic expression for $\partial\mathbf{\Delta}/\partial\mathbf{F}$ is

$$\frac{\partial\mathbf{\Delta}}{\partial\mathbf{F}} = \mathbf{\Phi}\mathbf{\Lambda}^{-1}\mathbf{\Phi}^T, \quad (23)$$

and the Local Sensitivity Matrix in Equation 5 can be rewritten as

$$\text{LSM} = \begin{bmatrix} \mathbf{I} & -\frac{\partial\mathbf{F}}{\partial\mathbf{q}}(\mathbf{\Phi}^T\mathbf{\Phi})^{-1}\mathbf{\Phi}^T \\ -\mathbf{\Phi}\mathbf{\Lambda}^{-1}\mathbf{\Phi}^T & \mathbf{I} \end{bmatrix}. \quad (24)$$

The remaining terms in the Global Sensitivity Equation are $\partial\mathbf{F}/\partial\mathbf{q}$ in the LSM, along with $\partial\mathbf{F}/\partial\mathbf{X}$ and $\partial\mathbf{\Delta}/\partial\mathbf{X}$ in the LSV. In this study, these terms are evaluated using finite difference approximations. However, clearly it is advisable to use analytic forms of these partial derivatives if such information is available. Current CFD solvers such as CFL3D.ADI [18] and SENSE [19], as well as CSM solvers including MSC/NASTRAN [20] and GENESIS [4], can provide some of the needed partial derivative terms.

4 Beam Example

A simple example problem is shown below to demonstrate the GSE/modal analysis method. Consider a cantilever beam of square cross section as shown in Figure 2. The vertical force on the tip of the beam acts in the positive z-direction with a magnitude that depends on the amount of deflection. The magnitude of the load, F , in units of pounds, is

$$F = 900 \text{ lb} - (2000 \text{ lb/ft}) \delta. \quad (25)$$

Using basic mechanics of materials, the vertical deflection at the tip of the beam is

$$\delta = \frac{FL^3}{3EI}, \quad (26)$$

where the length of the beam, L , is 15 ft, Young's modulus, E , is $2.3 \times 10^9 \text{ lb/ft}^2$ (for titanium alloy Ti-6Al-4V), and the area moment of inertia, I , is 0.00521 ft^4 ($h = w = 0.5 \text{ ft}$). The solution for F and δ to this set of coupled equations is $F = 757.725 \text{ lb}$ and $\delta = 0.071137 \text{ ft}$.

4.1 Exact Gradients

The total derivatives of F and δ with respect to an independent variable, e.g., beam length, may be found analytically as

$$\frac{dF}{dL} = \frac{-1.2 \times 10^7 EIL^2}{(3EI + 2000L^3)^2} \approx -23.9567 \text{ lb/ft}, \quad (27)$$

and

$$\frac{d\delta}{dL} = \frac{8100EIL^2}{(3EI + 2000L^3)^2} \approx 0.011978 \text{ lb/ft}. \quad (28)$$

The exact gradients also may be found using the GSE formulation. The GSE for this coupled system is

$$\begin{bmatrix} \mathbf{I} & -\frac{\partial F}{\partial \delta} \\ -\frac{\partial \delta}{\partial F} & \mathbf{I} \end{bmatrix} \begin{Bmatrix} \frac{dF}{dL} \\ \frac{d\delta}{dL} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial F}{\partial L} \\ \frac{\partial \delta}{\partial L} \end{Bmatrix}. \quad (29)$$

Substitute the following partial derivatives into the GSE

$$\frac{\partial F}{\partial \delta} = -2000, \quad (30)$$

$$\frac{\partial \delta}{\partial F} = \frac{L^3}{3EI}, \quad (31)$$

$$\frac{\partial F}{\partial L} = 0, \quad (32)$$

and,

$$\frac{\partial \delta}{\partial L} = \frac{FL^2}{EI}. \quad (33)$$

Solving for the unknown Total Sensitivity Vector terms gives

$$\frac{dF}{dL} = \frac{-6000FL^2}{3EI + 2000L^3} \approx -23.9567 \text{ lb/ft}, \quad (34)$$

and,

$$\frac{d\delta}{dL} = \frac{3FL^2}{3EI + 2000L^3} \approx 0.011978 \text{ lb/ft}, \quad (35)$$

which, as expected, are identical to the total derivative values computed explicitly.

4.2 Approximate Gradients Using GSE/Modal Analysis

Using the GSE/modal analysis approach described in Section 3, a four node, three element finite element model was created for the cantilever beam using the commercial CSM code GENESIS [4]. In this model, node points 1-4 are placed at 0.0, 5.0, 10.0 and 15.0 *ft*, respectively, with the applied load at node #4. The first three natural frequencies and mode shapes are listed in Table 1.

The partial derivatives in the GSE were calculated using nodes 2-4, since these were free to deflect. Note that the subscripts used below indicate node number. The partial derivatives of force with respect to deflection are

$$\frac{\partial F_i}{\partial \delta_j} = 0, \text{ for } i = 2, \dots, 4 \text{ and } j = 2, 3, \quad (36)$$

and

$$\frac{\partial F_4}{\partial \delta_4} = -2000. \quad (37)$$

The remaining partial derivatives in the GSE are

$$\frac{\partial \delta}{\partial F} = \Phi \Lambda^{-1} \Phi, \quad (38)$$

$$\frac{\partial F_{2-4}}{\partial L} = \{0, 0, 0\}, \quad (39)$$

and,

$$\frac{\partial \delta_{2-4}}{\partial L} = \{0.00212893, 0.007618, 0.0143702\}. \quad (40)$$

Note that the term $\partial \delta / \partial L$ was computed using a first-order, forward-step, finite difference approximation with a step size of 1.0% ($\Delta L = 0.15$).

Solving the GSE for the terms in the Total Sensitivity Vector yields

$$\frac{dF_4}{dL} \approx -24.2028 \text{ lb/ft}, \quad (41)$$

and

$$\frac{d\delta_4}{dL} \approx 0.012101 \text{ lb/ft}, \quad (42)$$

These estimates for the total derivatives dF/dL and $d\delta/dL$ agree to within 3.0 percent of the exact values for the total derivatives.

5 Aircraft Analysis Example

The GSE/modal analysis method is applied to an aircraft analysis and design problem which involves computationally expensive CFD and CSM codes. High fidelity static aeroelastic analysis is performed for a supersonic transport aircraft (see Figure 3) at Mach 2.4, 1.0*g*, cruise conditions, at an angle-of-attack, α , of 3.5°. The application of the GSE/modal analysis methods enables the efficient evaluation of gradients of the aerodynamic lift, drag, and pitching moment coefficients (C_L , C_D , and C_{M_o}) with respect to any of the design parameters. These gradients take into account the aero-structural interaction in the aeroelastic system.

5.1 Aircraft Parametric Model

A parametric wing/fuselage model of a supersonic transport was developed for this study and is detailed in Reference [21]. This model contains 64 parameters which define the outer mold line (OML) of the wing (i.e., planform and shape parameters). Six of the parameters which define the wing planform are shown in Figure 4. Other parameters define the thickness,

camber, and dihedral at various spanwise locations on the wing.

In addition to the 64 OML parameters, there are 40 parameters that define the structural model, including the thickness of the wing skin panels and the sizes of various rib and spar structural elements.

5.2 Aircraft CFD Model

Surface and volume grids for CFD analysis are generated based on the OML parameters using the code CSCMDO (Coordinate and Sensitivity Calculator for Multi-disciplinary Design Optimization) [22]. The output from CSCMDO is a structured grid with a C-O discretization scheme and dimensions of $121 \times 41 \times 61$, in the streamwise, circumferential, and surface normal directions, respectively. The volume grid is divided into two blocks with the interface splitting the wing into upper and lower surfaces. There are approximately 300,000 grid points in the model (see Figure 5).

The CFD code CFL3D [3] was provided for this study by the Aerodynamics and Acoustics Methods Branch at NASA Langley Research Center. CFL3D is a time-dependent, Reynolds-averaged, thin-layer Navier-Stokes flow solver for use with two- or three-dimensional structured grids. Both mesh sequencing and multigrid techniques are available in CFL3D for convergence acceleration. In this study, CFL3D is used to resolve the inviscid, supersonic flow around the aircraft configuration. Nominal cruise conditions are Mach 2.4, $1.0g$ load factor, 3.5° angle-of-attack (with respect to the fuselage centerline), and an altitude of 63,000 *ft*. An initial flow solution with CFL3D starting from a uniform flow field requires approximately 60 CPU minutes on an SGI workstation with a 250 *MHz*, IP27 R10000 processor. Subsequent analyses require approximately 30 CPU minutes through the use of the restart capability in CFL3D.

5.3 Aircraft CSM Model

A finite element model of the aircraft structure is generated using the 64 OML parameters and the 40 structural parameters with the aid of software developed by Balabanov [23] for related research involving supersonic transport configurations. The wing/fuselage model of the aircraft is comprised of a fixed number and topology of spar and rib elements, along with wing skin elements (Figure 6). The layout of the structural elements is based on the OML parameters, whereas the size (e.g., thickness and area)

of the finite elements is specified through the 40 structural parameters.

The finite element model of the supersonic transport configuration has 226 nodes and 1130 elements with a total of 1254 degrees-of-freedom. Note that due to structural symmetry only the starboard portion of the model is constructed. The FE model contains triangular membrane elements for the fuselage and wing skins, along with rod elements for the spar cap and rib caps. The spar and rib webs are modeled with a combination of shear panels and rod elements. The material for all structural elements is titanium alloy Ti-6Al-4V.

The CSM solver GENESIS [4] is used in this study to perform linear structural analysis and modal analysis of the aircraft model. The computational expense for a single GENESIS analysis is approximately one CPU minute on an SGI workstation.

5.4 Loads Transfer and Structural Deformation

External aerodynamic loads for the GENESIS model are obtained using the CFD-CSM loads transfer software code FASIT (Fluids and Structures Interface Toolkit) developed by Smith et al [24, 25], and currently maintained by the Air Force Research Laboratory at Wright-Patterson Air Force Base. FASIT provides a suite of interpolation schemes for transferring the aerodynamic loads from the CFD model to the CSM model. The thin plate spline method of Duchon [26] was used in this study, as recommended in the FASIT User's Manual [25]. The loads transferred from FASIT to the CSM surface grid are written in the NASTRAN Bulk Data format. Since this NASTRAN format is compatible with the GENESIS input format, no translation was needed between FASIT and GENESIS. The computational expense of running FASIT is negligible (less than 10 CPU seconds).

Displacement of the nodes in the CSM model and the first 16 mode shapes are computed using GENESIS. A series of Mathematica [27] programs are used to calculate the mode shape scale factors (see Equation 13) needed to represent the node displacements as a superposition of the 16 mode shapes. The rationale for choosing 16 mode shapes is described in Section 5.6 below. Additional Mathematica programs calculate the changes in the 64 OML parameters due to structural deformation. A new CFD grid is then generated from the updated list of 64 OML parameters.

5.5 Aeroelastic Analysis

Aeroelastic analysis is performed by coupling the CFD, CSM, and interpolation codes as depicted in Figure 7. The box labeled “Geometry Manipulator” in Figure 7 contains the software used to generate the CFD and CSM parametric models, including the various Mathematica programs described above. The CFD, CSM, and interpolation software used in the study is loosely coupled using UNIX shell scripts and some simple file manipulation codes. More detail on the software coupling methods is provided in Reference [21].

Due to the nonlinearity of the aero/structural interaction, the aeroelastic analysis involves an iterative procedure whereby the aerodynamic and structural analyses are performed repeatedly until both the aerodynamic loads and the structural deflections reach convergence. The convergence criterion used in this loop is based on the z-direction displacement of the leading edge of the wing tip. If the difference in this z-displacement value between two successive passes through the loop is less than 0.05 *ft*, then the aeroelastic analysis is considered to be converged.

To reduce the oscillations that occur in this under-damped system, a constant factor under-relaxation method is used to accelerate convergence. This under-relaxation scheme follows the approach of Chipman et al [28] and Tzong et al [29]. A value of 0.7 for the relaxation parameter is used for all aeroelastic calculations conducted in this study. The effect of this damping parameter is shown in the convergence history plot shown in Figure 8. This figure shows the displacement history of the wing tip versus the number of passes through the aeroelastic loop in Figure 7. Typically, convergence is obtained in 6-10 iterations (2.7 CPU hours) when under-relaxation is used as compared to 19 iterations (6.1 CPU hours) without the relaxation method.

Results for an aeroelastic analysis at the nominal Mach 2.4, 1.0*g* cruise conditions are shown in Figures 9 and 10. In Figure 9 the final deformed wing is shown in comparison to the solid outline of the undeformed wing. Note that the z-axis in this figure has been scaled to exaggerate the wing deflection for easier viewing. Figure 10 shows the aeroelastic deformation of the aircraft wing at the wing tip, wing break ($y = 33.7$ *ft*), and wing root. At the 1.0*g* cruise conditions the z-direction (upward) wing tip deflection is approximately 1.0 *ft*.

5.6 Mode Shape Selection Criteria

For this study 16 mode shapes were found to be more than sufficient to represent the structural de-

formation of the aircraft finite element model. This was demonstrated in a convergence study in which the number of modes used in the aeroelastic analysis procedure was varied from 1-16. Figure 11 shows these results as compared to an aeroelastic analysis performed using the exact structural deformations, i.e., without introducing the superposition of mode shapes. The correct final wing position was obtained if at least four mode shapes were used in the linear superposition. All of the aeroelastic analysis cases shown in Figure 11 were started from the undeflected aircraft shape with identical initial conditions.

6 Aircraft Sensitivity Analysis

Once a converged aeroelastic analysis was obtained for the aircraft model, the GSE method was used to estimate gradients of the coupled system with respect to perturbations in the independent design parameters. For this study, gradients of C_L , C_D , and C_{M_o} were computed with respect to three different design parameters: (1) $X = \alpha$, i.e., the angle-of-attack at cruise, (2) $X = (t/c)_{\text{break}}$, the thickness-to-chord ratio at the wing leading edge break location (see Figure 4), and (3) $X = \Lambda_{LE}$, the inboard leading edge wing sweep angle.

Calculating gradients for this coupled aero-structural system involves computing the terms in the Global Sensitivity Equation with the modification to the Local Sensitivity Matrix shown in Equation 24. Solving the GSE yields the total derivatives $d\mathbf{F}/dX$ and $d\mathbf{\Delta}/dX$, which estimate the change in the aerodynamic loads and structural deflections due to a perturbation in the parameter X . These gradient vectors are then used to compute the gradients for C_L , C_D , and C_{M_o} . This is possible since the aerodynamic coefficients have a functional form identical to that of the aerodynamic loads. That is,

$$C_L = C_L(X, \mathbf{\Delta}), \quad (43)$$

along with similar expressions for C_D and C_{M_o} . Differentiating Equation 43 with respect to the parameter X , yields

$$\frac{dC_L}{dX} = \frac{\partial C_L}{\partial X} + \frac{\partial C_L}{\partial \mathbf{\Delta}} \frac{d\mathbf{\Delta}}{dX}. \quad (44)$$

Following the approach shown in Equations 14-16 yields

$$\frac{dC_L}{dX} = \frac{\partial C_L}{\partial X} + \frac{\partial C_L}{\partial \mathbf{q}} (\mathbf{\Phi}^T \mathbf{\Phi})^{-1} \mathbf{\Phi}^T \frac{d\mathbf{\Delta}}{dX}. \quad (45)$$

Thus, the total derivatives for C_L , C_D , and C_{M_o} can be computed using the total derivative $d\mathbf{\Delta}/dX$

obtained from the GSE. These total derivatives for C_L , C_D , and C_{M_o} may then be used in Taylor Series approximations of the form

$$C_L(X + \delta X) \approx C_L(X) + \frac{dC_L}{dX}\delta X. \quad (46)$$

This Taylor Series approximation is an inexpensive technique for estimating the changes in C_L , C_D , and C_{M_o} due to small perturbations of X , in contrast to an expensive aeroelastic analysis for $X + \delta X$.

6.1 Interdisciplinary Coupling Terms

Computing the interdisciplinary coupling terms in Equation 24 involves the mode shape and eigenvalue data from the finite element solver GENESIS. Some of this data is listed in Table 2 which includes the first 16 natural frequencies and eigenvalues for the supersonic transport model, along with the mode shape scale factors computed using Equation 13. Note that this eigenvalue analysis data is obtained from the undeformed finite element model of the aircraft, i.e., without application of any loads. The matrix of mode shapes, Φ , has 226 rows and 16 columns. That is, each row corresponds to one of the nodes in the finite element model, and each column corresponds to a different mode shape.

The partial derivative term $\partial \mathbf{F} / \partial \mathbf{q}$ in the Local Sensitivity Matrix is calculated by perturbing each mode shape coefficient by 20 percent, and then performing a CFD analysis for each of the new aircraft shapes. The 16 CFD analyses are performed using coarse grained parallel computing on an SGI Origin 2000 computer. That is, the CFD analyses are executed simultaneously, with each CFD analysis assigned to a separate processor on the parallel computer. The SGI Origin 2000 computers at NASA Langley Research Center and NASA Ames Research Center were used in this study. The performance of this parallel computing approach is shown in Figure 12, where “speedup” is defined as the ratio of the total serial computational time to the parallel execution time. The discrepancy between ideal and actual speedup is due primarily to file transfer operations between the processors and disk storage. With 16 simultaneous CFD analyses on the NASA Langley Origin 2000, an aggregate parallel performance of 380 MFLOPS was achieved. By using parallel computing, 16 CFD analyses can be completed in about the same wall-clock time as is needed for a single CFD analysis.

6.2 Local Sensitivity Terms

The partial derivative terms in the Local Sensitivity Vector on the right side of Equation 5 must be computed for each design parameter X , before solving the GSE. Some CFD and CSM solvers may provide these partial derivatives based on analytic differentiation of the underlying CFD and CSM state equations. If analytic expressions for the partial derivatives are unavailable, finite difference approximations may be used to estimate the partial derivatives. The finite difference approach is used in this study.

6.3 Global Sensitivity Calculations

6.3.1 Angle-of-Attack Sensitivity

For the angle-of-attack sensitivity calculations the partial derivative term $\partial \mathbf{F} / \partial X = \partial \mathbf{F} / \partial \alpha$ was computed using finite differences. The aerodynamic loads were obtained from the initial aeroelastic analysis results at $\alpha = 3.5^\circ$, and from one additional CFD analysis at $\alpha = 4.0^\circ$.

The nodal displacements from the CSM code are not explicitly dependent on angle-of-attack. Therefore,

$$\frac{\partial \Delta}{\partial X} = \frac{\partial \Delta}{\partial \alpha} = \{0\}. \quad (47)$$

The GSE was then solved for the unknown total derivatives $d\mathbf{F}/d\alpha$ and $d\Delta/d\alpha$. Gradients for C_L , C_D , and C_{M_o} were computed using Equation 45, with the terms $\partial C_L / \partial \mathbf{q}$, $\partial C_D / \partial \mathbf{q}$, and $\partial C_{M_o} / \partial \mathbf{q}$ computed using the results from the 16 CFD analyses for the mode shape perturbations. The gradients of the aerodynamic coefficients are listed in Table 3. For comparison, the same gradients were computed using finite differences, with an additional expensive aeroelastic analysis at $\alpha = 4.0^\circ$. Good agreement is obtained between the GSE-based gradients and the finite difference gradients.

Taylor Series approximations for C_L , C_D , and C_{M_o} at $\alpha = 4.0^\circ$ are listed in Table 4, along with the exact values for C_L , C_D , and C_{M_o} computed in the expensive aeroelastic analysis at $\alpha = 4.0^\circ$. Good agreement is obtained between the approximate and exact values of the aerodynamic coefficients.

6.3.2 t/c Ratio Sensitivity

For the thickness-to-chord ratio sensitivity calculations the nominal value of the t/c ratio at the wing break was 2.36 percent, and the perturbed value of the t/c ratio was 2.60 percent. The partial derivative terms $\partial \mathbf{F} / \partial (t/c)$ and $\partial \Delta / \partial (t/c)$ were computed

using finite difference approximations. The aerodynamic loads for $t/c = 2.60$ percent were obtained with a single additional CFD analysis. Similarly, the structural deflections for $t/c = 2.60$ percent were obtained from one additional CSM analysis.

Since the terms in the Local Sensitivity Matrix do not change with respect to perturbations in the design variables there was no need to recompute the mode shape partial derivative term $\partial \mathbf{F}/\partial \mathbf{q}$. Thus, the previously computed LSM was reused when solving the GSE for this t/c sensitivity analysis case.

The total derivatives $d\mathbf{F}/d(t/c)$ and $d\Delta/d(t/c)$ were computed by solving the GSE, and gradients for the aerodynamic coefficients were estimated using Equation 45. These values are listed in Table 5 and show good agreement with gradients computed using a finite difference approximation based on the expensive aeroelastic analysis results.

Taylor Series approximations for C_L , C_D , and C_{M_o} for a wing break t/c ratio of 2.60 percent are listed in Table 6. These approximations are in good agreement with aerodynamic coefficients obtained from the expensive aeroelastic analysis with the wing break t/c ratio of 2.60 percent.

6.3.3 Wing Sweep Angle Sensitivity

The sensitivity of the aerodynamic coefficients to perturbations in the inboard wing sweep angle were computed using the GSE method. The nominal wing sweep for the aircraft was 74.0° , and a perturbation of $+2.0^\circ$ was used in this study. A single CFD analysis for a wing sweep of 76.0° was used to create a finite difference approximation for $\partial \mathbf{F}/\partial \Lambda_{LE}$. Similarly, the results from a single CSM analysis were used to estimate $\partial \Delta/\partial \Lambda_{LE}$. As before, the LSM does not change with respect to the aircraft design parameters so it is reused in this sensitivity analysis.

After solving the GSE for the total sensitivity terms $d\mathbf{F}/d\Lambda_{LE}$ and $d\Delta/d\Lambda_{LE}$, gradients of the aerodynamic coefficients were estimated using Equation 45. These gradients are listed in Table 7. Taylor Series approximations for C_L , C_D , and C_{M_o} for a sweep angle of 76.0° are listed in Table 8. Both the gradients and the Taylor Series approximations obtained from the GSE method are in good agreement with values obtained from expensive aeroelastic analyses for an aircraft with an inboard wing sweep angle of 76.0° .

7 Computational Savings with the GSE Approach

The advantage of computing gradients with the GSE/modal analysis approach rather than a pure finite difference approach is illustrated in Figure 13. This plot shows the number of CFD evaluations needed to compute gradients of the coupled aero-structural system, versus the number of independent design parameters. Here, the number of CFD evaluation is used as a cost metric since a CFD evaluation is about 30 times more expensive than any other portion of the aeroelastic analysis process.

In the pure finite difference approach, computing the gradients for each new parameter requires about 10 CFD analyses. This cost stems from the need to perform an aeroelastic analysis for each perturbation of a design parameter. Thus, the cost of the pure finite difference approach is linear with respect to the number of independent variables and is given by the expression

$$COST_{FD} = 10n_v, \quad (48)$$

where n_v is the number of independent variables.

In the GSE/modal analysis approach there is an initial cost of 16 CFD evaluations, i.e., one CFD evaluation for each mode shape coefficient perturbation in the partial derivative term $\partial \mathbf{F}/\partial \mathbf{q}$. However, after this initial cost only one new CFD evaluation is needed for each design parameter. Thus, the GSE/modal analysis approach quickly becomes more attractive, from a computational standpoint, if gradients are needed for more than two design parameters. The cost of the GSE/modal analysis approach is

$$COST_{GSE/Modal} = n_v + 16. \quad (49)$$

Note that the GSE approach without modal analysis would require a separate CFD evaluation for each of the elements of the partial derivative term $\partial \mathbf{F}/\partial \Delta$. With the models used in this aeroelastic analysis, the cost of a GSE-alone approach would be

$$COST_{GSE-alone} = n_v + 226, \quad (50)$$

where 226 is the number of nodes in the finite element model. For larger, more realistic finite element models the value 226 would grow to be $\mathcal{O}(10^3 - 10^4)$.

In theory, the application of coarse grained parallel computing renders the wall-clock time identical for both the GSE/modal analysis method and the GSE-alone method. However, the burden of file management and serial file input/output make the GSE-alone approach unattractive, even when a parallel computer having hundreds or thousands of processors is available.

Although not attempted in this study, the GSE/modal analysis approach offers significant opportunities for multi-level parallelization. For example, fine grained parallel computing could be used to perform a CFD analysis on an aircraft model having several million grid points. With such a large grid the computational domain may be subdivided into hundreds of zones, each of which is assigned to a separate processor on a parallel computer. The coarse grained GSE/modal analysis approach would then provide an additional level of parallelism. As an illustration of this multi-level paradigm, consider a large CFD grid having 100 zones, each of which executes simultaneously on a separate processor of a parallel computer. The GSE/modal analysis approach would replicate this 100-zone grid 16 times, i.e., once for each perturbation of a mode shape coefficient. Such a strategy would efficiently utilize 1600 processors. Furthermore, this multi-level strategy readily accommodates increases in the number of zones (fine grained scalability) and in the number of mode shapes (coarse grained scalability).

8 Conclusions

A method based on the Global Sensitivity Equations and modal analysis has been developed to calculate gradients of aerodynamic force and moment coefficients for an aeroelastic aircraft model. The Global Sensitivity Equations capture the the aero-structural coupling in the supersonic transport aircraft model examined in this study. Modal analysis is employed to reduce the coupling bandwidth between the aerodynamic and structural models. Coarse grained parallel computing is used with the high fidelity computational fluid dynamics solver, CFL3D, for the efficient calculation of partial derivative terms in the GSE/modal analysis approach.

A sensitivity analysis was performed for the supersonic transport aircraft model at nominal Mach 2.4 cruise conditions. The GSE/modal analysis approach was used to estimate the gradients of C_L , C_D and C_{M_o} with respect to variations in three of the aircraft design parameters. Good agreement was obtained between the GSE-based gradients and finite difference-based gradients of the aerodynamic coefficients. In addition, approximations for C_L , C_D and C_{M_o} were computed with the GSE/modal analysis approach for small perturbations in the three aircraft design parameters. These GSE-based approximations were in good agreement with exact values for C_L , C_D and C_{M_o} computed using the high fidelity aerodynamic and structural models.

A comparison of the computational expense for the GSE/modal analysis method and for the brute-force finite difference method demonstrated the advantage of using the GSE/modal analysis approach if gradients are needed for more than two aircraft design parameters. Thus, the initial cost of the GSE/modal analysis approach is quickly recovered in a realistic aircraft sensitivity analysis which may involve tens or hundreds of independent parameters.

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Table 1: Natural frequencies, eigenvalues, and eigenvectors for the four node beam finite element model.

	Mode 1	Mode 2	Mode 3
Frequency	5.86 Hz	36.77 Hz	103.69 Hz
Eigenvalue	1354.0	53388.0	424454.6
Eigenvector Components			
Node 1	0.0	0.0	0.0
Node 2	0.058156	-0.208162	0.262108
Node 3	0.192160	-0.149674	-0.230501
Node 4	0.351349	0.352866	0.351239

Table 2: Natural frequencies, eigenvalues, and scale factors for the first 16 modes of the aircraft finite element model.

Mode	Frequency (Hz)	Eigenvalue	Scale Factor
1	1.75	121.20	6.7740953
2	4.37	754.61	-1.9055302
3	7.05	1961.95	0.5120722
4	9.21	3350.76	0.0584607
5	10.94	4727.34	0.0303337
6	14.82	8675.36	-0.0675731
7	16.52	10768.64	-0.1006403
8	19.67	15277.35	0.0534937
9	24.42	23533.04	-0.0464125
10	25.33	25332.50	0.0233427
11	28.71	32539.54	-0.0528983
12	30.93	37765.35	-0.0113462
13	33.48	44241.15	0.0265745
14	34.47	46894.51	-0.0036646
15	37.59	55771.28	-0.0068915
16	41.37	67567.14	-0.0055217

Table 3: Gradients of C_L , C_D , and C_{M_o} due to perturbations in cruise angle-of-attack ($X = \alpha$). The finite difference values are computed from aeroelastic analyses at $\alpha = 3.5^\circ$ (nominal condition) and $\alpha = 4.0^\circ$. The gradients are in units of $1/deg$.

	GSE	Finite Difference
dC_L/dX	0.02432	0.02404
dC_D/dX	0.002486	0.002408
dC_{M_o}/dX	-0.02763	-0.02707

Table 4: Aerodynamic coefficients at an angle-of-attack of 4.0° . The Taylor Series approximations use gradients from the GSE.

	Taylor Series with GSE	New Aeroelastic Analysis
C_L	0.090385	0.089027
C_D	0.006231	0.006129
C_{M_o}	-0.101000	-0.099099

Table 5: Gradients of C_L , C_D , and C_{M_o} due to perturbations in t/c ratio at the wing break. The finite difference values are computed from aeroelastic analyses at $t/c = 2.36\%$ (nominal value) and $t/c = 2.60\%$. The gradients are nondimensional.

	GSE	Finite Difference
dC_L/dX	6.317×10^{-4}	9.958×10^{-4}
dC_D/dX	3.835×10^{-4}	4.033×10^{-4}
dC_{M_o}/dX	-0.929×10^{-3}	-1.446×10^{-3}

Table 6: Aerodynamic coefficients for a new t/c ratio of 2.6 percent. The Taylor Series approximations use gradients from the GSE.

	Taylor Series with GSE	New Aeroelastic Analysis
C_L	0.077207	0.077295
C_D	0.005017	0.005022
C_{M_o}	-0.085789	-0.085913

Table 7: Gradients of C_L , C_D , and C_{M_o} due to perturbations in the leading edge sweep angle ($X = \Lambda_{LE}$). The finite difference values are computed from aeroelastic analyses at $\Lambda_{LE} = 74.0^\circ$ (nominal condition) and $\Lambda_{LE} = 76.0^\circ$. The gradients are in units of $1/\text{deg}$.

	GSE	Finite Difference
dC_L/dX	-0.003385	-0.004111
dC_D/dX	-0.000224	-0.000266
dC_{M_o}/dX	0.003359	0.004355

Table 8: Aerodynamic coefficients for a leading edge wing sweep angle of 76.0° . The Taylor Series approximations use gradients from the GSE.

	Taylor Series with GSE	New Aeroelastic Analysis
C_L	0.070285	0.068835
C_D	0.004478	0.004393
C_{M_o}	-0.078848	-0.076857

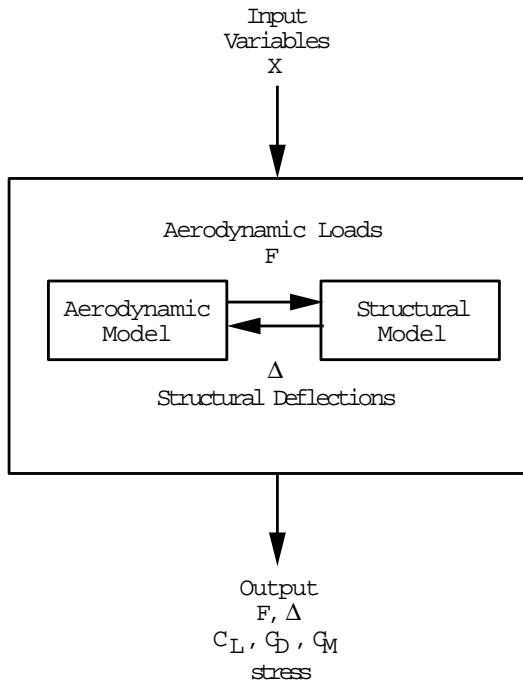


Figure 1: Depiction of a coupled aero-structural system.

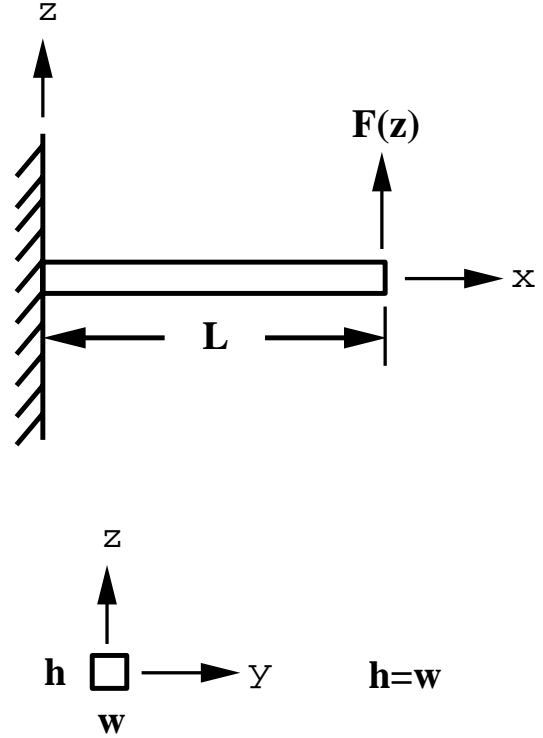


Figure 2: Sample problem of a cantilever beam subject to a tip load, where the load magnitude depends on the tip displacement.

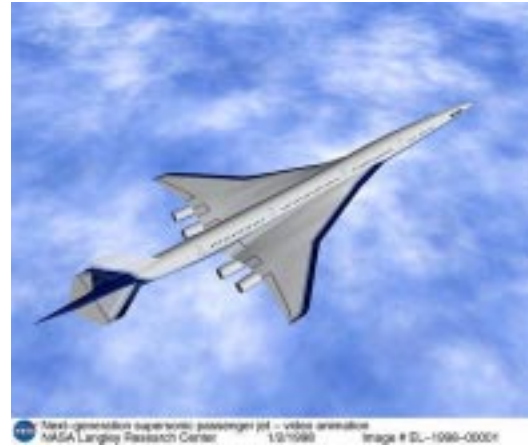


Figure 3: Supersonic transport aircraft.

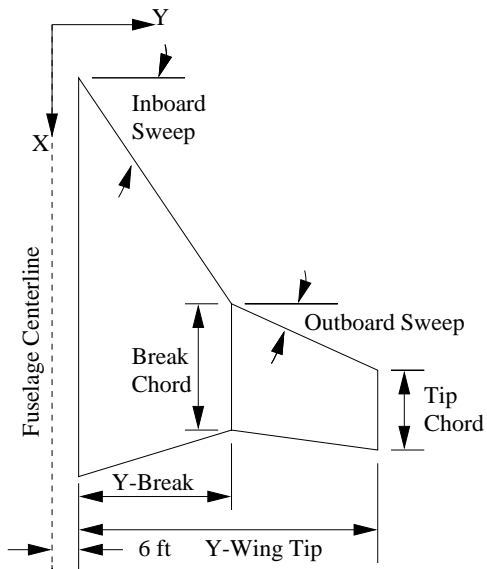


Figure 4: Planform variables for the aircraft wing.

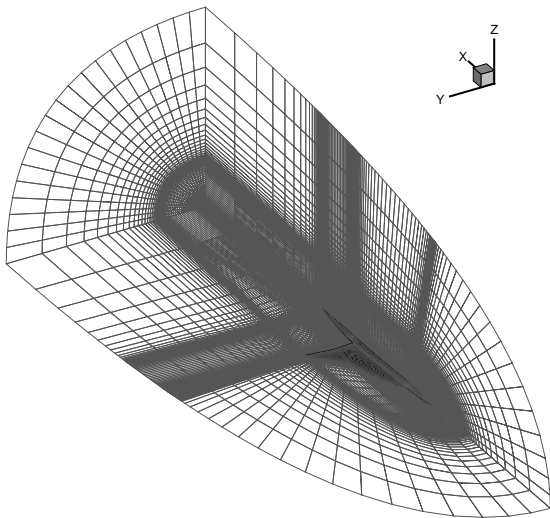


Figure 5: A view of one block in the aerodynamic model of the supersonic transport. This grid shows the starboard wing, the $x-z$ plane of symmetry, and the exit plane.

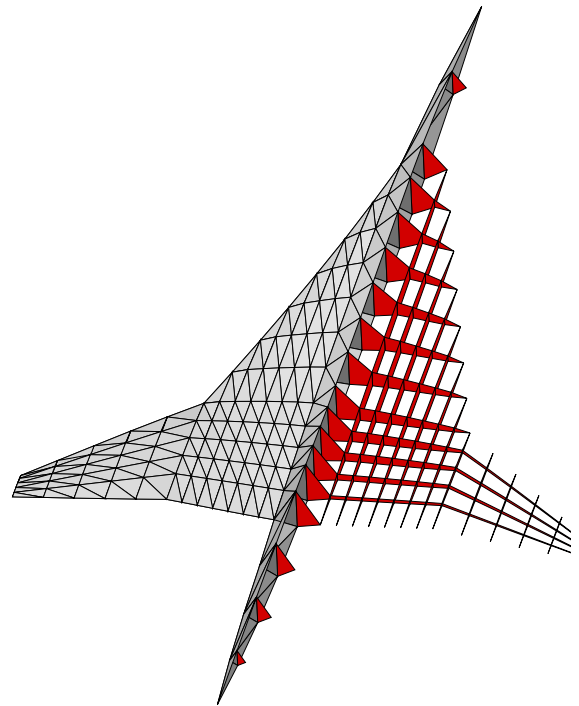


Figure 6: The structural model of the aircraft showing the wing skin elements (port) and the rib/spar elements (starboard).

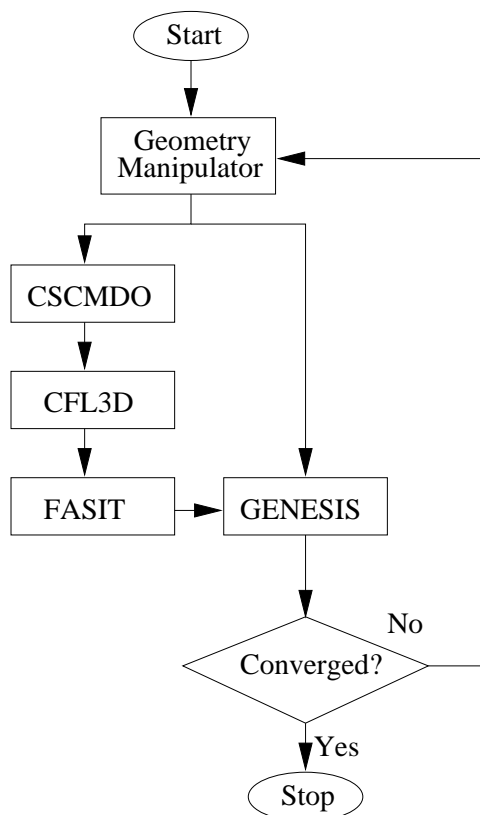


Figure 7: The arrangement of software used to perform static aeroelastic analysis.

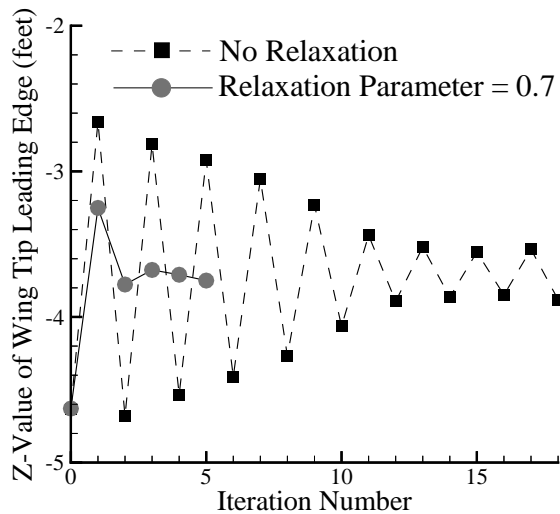


Figure 8: Convergence history of the aeroelastic analysis with and without relaxation.

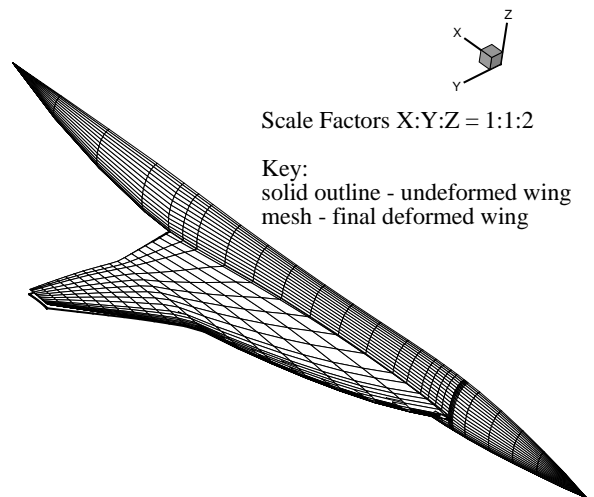


Figure 9: Orthographic view of the deformed wing (mesh) and the undeformed wing (solid outline). Note the X:Y:Z scaling of 1:1:2 used to show the wing deformation.

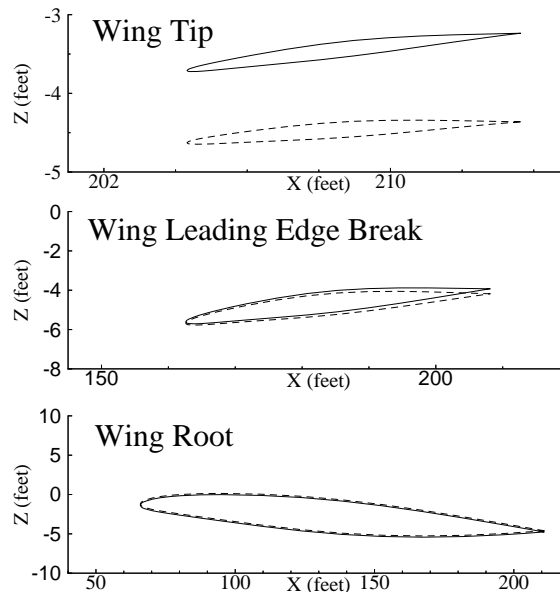


Figure 10: Airfoil sections at the (top-bottom) wing tip, wing break, and wing root for the initial undeformed (dashed) and final deformed (solid) wing shapes.

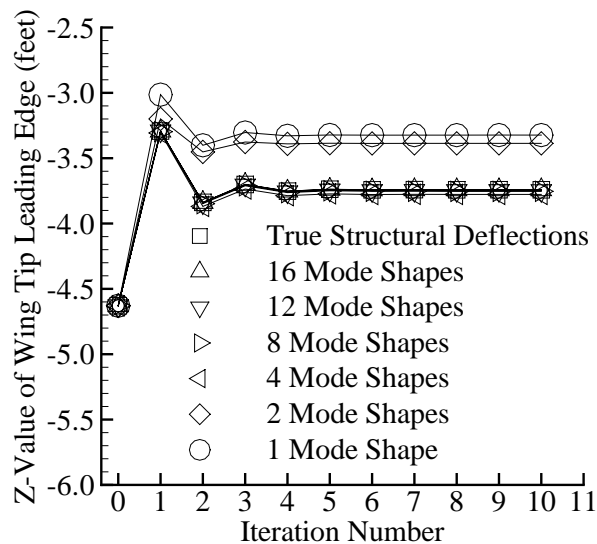


Figure 11: Convergence study showing the effect of varying the number of mode shapes used to approximate the structural deformation of the aircraft finite element model.

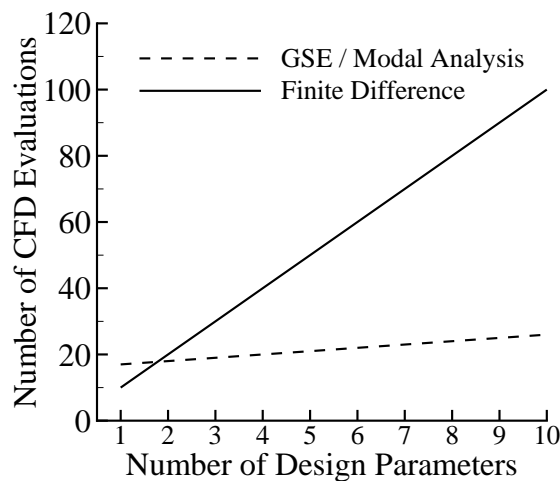


Figure 13: Cost of computing total sensitivity derivatives using finite differences (solid) and the GSE/modal analysis approach (dashed).

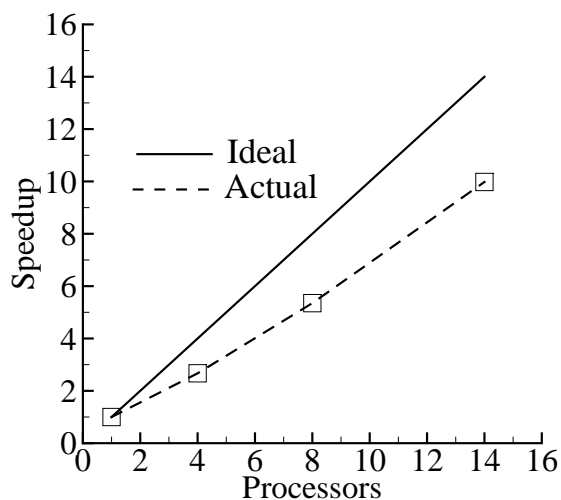


Figure 12: Speedup obtained using coarse grained parallel execution of CFL3D on the NASA Langley SGI Origin 2000.

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13. ABSTRACT (Maximum 200 words) A novel method has been developed for calculating gradients of aerodynamic force and moment coefficients for an aeroelastic aircraft model. This method uses the Global Sensitivity Equations (GSE) to account for the aero-structural coupling, and a reduced-order modal analysis approach to condense the coupling bandwidth between the aerodynamic and structural models. Parallel computing is applied to reduce the computational expense of the numerous high fidelity aerodynamic analyses needed for the coupled aero-structural system. Good agreement is obtained between aerodynamic force and moment gradients computed with the GSE/modal analysis approach and the same quantities computed using brute-force, computationally expensive, finite difference approximations. A comparison between the computational expense of the GSE/modal analysis method and a pure finite difference approach is presented. These results show that the GSE/modal analysis approach is the more computationally efficient technique if sensitivity analysis is to be performed for two or more aircraft design parameters.				
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